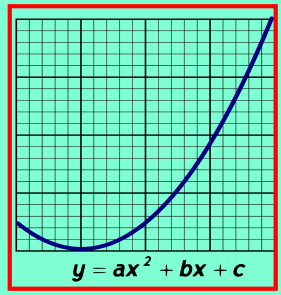
Math 125

Spring 2021

Lecture 15



Class QZ 10

Solve by Cramer's rule

$$\begin{cases} 5x + 3y = 1 \\ 2x - y = 7 \end{cases}$$

$$D = \begin{vmatrix} 5 & 3 \\ 2 & -1 \end{vmatrix} = 5(-1) - 2(3) = \boxed{-11}$$

$$D_x = \begin{vmatrix} 1 & 3 \\ 7 & -1 \end{vmatrix} = 1(-1) - 7(3) = \boxed{-22}$$

$$D_y = \begin{vmatrix} 5 & 1 \\ 2 & 7 \end{vmatrix} = 5(7) - 2(1) = \boxed{33}$$

$$x = \frac{D_x}{D} = \frac{-22}{-11} = 2$$

$$y = \frac{D_y}{D} = \frac{33}{-11} = -3$$

$$\boxed{\text{Final Ans: } (2, -3)}$$

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Solve $\begin{cases} 3x + 7y - 15z = -12 \\ x + 2y - 4z = -3 \\ -4x - 6y + 15z = 16 \end{cases}$ by Matrix Method.

$$\left[\begin{array}{ccc|c} 3 & 7 & -15 & -12 \\ 1 & 2 & -4 & -3 \\ -4 & -6 & 15 & 16 \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -3 \\ 3 & 7 & -15 & -12 \\ -4 & -6 & 15 & 16 \end{array}\right]$$

$(-3)R1 + R2 \to R2$

$(4)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -3 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{array}\right]$$

$(-2)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -3 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 5 & 10 \end{array}\right]$$

$R3 \div (5) \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -3 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & -3 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Gaussian Elimination

All Zero's

All 1's.

$x + 2y - 4z = -3$

$y - 3z = -3$

$z = 2$

$y - 3(2) = -3$

$y - 6 = -3$

$\boxed{y = 3}$

$x + 2(3) - 4(2) = -3$

$x + 6 - 8 = -3$

$\boxed{x = -1}$

Final Ans

$(-1, 3, 2)$

Solve by Matrix Method:

$$\begin{cases} 2x + 7y + z = 14 \\ x + 3y - z = 2 \\ x + 7y + 12z = 45 \end{cases}$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{ccc|c} 2 & 7 & 1 & 14 \\ 1 & 3 & -1 & 2 \\ 1 & 7 & 12 & 45 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 2 & 7 & 1 & 14 \\ 1 & 7 & 12 & 45 \end{array}\right]$$

$(-2)R1 + R2 \to R2$

$(-)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 1 & 3 & 10 \\ 0 & 4 & 13 & 43 \end{array}\right]$$

$(-4)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 1 & 3 & 10 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Gaussian Elimination

$x + 3y - z = 2$ $\quad x = 2$

$y + 3z = 10$ $\quad y = 1$

$z = 3$

$(2, 1, 3)$

Gauss-Jordan Elimination

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 2 \\ 0 & 1 & 3 & 10 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$(-3)R3 + R2 \to R2$

$R3 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$(-3)R2 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x = 2$

$y = 1$

$z = 3$

Final Ans

$(2, 1, 3)$

Solve by matrix Method:

$$\begin{cases} 2x + 7y + 11z = 11 \\ x + 2y + 8z = 14 \\ x + 3y + 6z = 8 \end{cases}$$

Augmented Matrix

$R2 \rightleftarrows R1$

$$\left[\begin{array}{ccc|c} 2 & 7 & 11 & 11 \\ 1 & 2 & 8 & 14 \\ 1 & 3 & 6 & 8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 14 \\ 2 & 7 & 11 & 11 \\ 1 & 3 & 6 & 8 \end{array}\right]$$

$(-2)R1 + R2 \to R2$

$(-)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 14 \\ 0 & 3 & -5 & -17 \\ 0 & 1 & -2 & -6 \end{array}\right]$$

$R2 \rightleftarrows R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 14 \\ 0 & 1 & -2 & -6 \\ 0 & 3 & -5 & -17 \end{array}\right]$$

$(-3)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 14 \\ 0 & 1 & -2 & -6 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 8 & 14 \\ 0 & 1 & -2 & -6 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$(2)R3 + R2 \to R2$

$(-8)R3 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 6 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$(-2)R2 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 14 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} x = 14 \\ y = -4 \\ z = 1 \end{array} \Rightarrow$$

Final Ans

$(14, -4, 1)$

Graph of the equation $y = ax^2 + bx + c$ Contains $(1,4)$, $(-1,10)$, and $(2,7)$.
Use matrix method to find the eqn.

Point $(1,4)$

$x = 1$, $y = 4$ $\Rightarrow 4 = a(1)^2 + b(1) + c \Rightarrow \boxed{a + b + c = 4}$

Point $(-1,10)$

$x = -1$, $y = 10$ $\Rightarrow 10 = a(-1)^2 + b(-1) + c \Rightarrow \boxed{a - b + c = 10}$

Point $(2,7)$

$x = 2$, $y = 7$ $\Rightarrow 7 = a(2)^2 + b(2) + c \Rightarrow \boxed{4a + 2b + c = 7}$

$$\begin{cases} a + b + c = 4 \\ a - b + c = 10 \\ 4a + 2b + c = 7 \end{cases} \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 10 \\ 4 & 2 & 1 & 7 \end{array}\right]$$

$(-1)R1 + R2 \rightarrow R2$
$(-4)R1 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & 0 & 6 \\ 0 & -2 & -3 & -9 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -2 & 0 & 6 \\ 0 & -2 & -3 & -9 \end{array}\right]$$

$R2 \div (-2) \rightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & -2 & -3 & -9 \end{array}\right]$$

$(2)R2 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & -3 & -15 \end{array}\right]$$

$y = ax^2 + bx + c$

$R3 \div (-3) \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$a + b + c = 4$

$\boxed{b = -3}$

$\boxed{c = 5}$

$a - 3 + 5 = 4$

$a + 2 = 4$

$\boxed{a = 2}$

$\boxed{y = 2x^2 - 3x + 5}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$(-1)R3 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$(-1)R2 + R1 \to R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad \begin{array}{l} a=2 \\ b=-3 \\ c=5 \end{array}$$

$$y = ax^2 + bx + c$$

$$\boxed{y = 2x^2 - 3x + 5}$$

Lisa has 20 Coins.

Dimes, Nickels, and Pennies only.

She has \$1.32 → 132¢

# of Nickels is the Same as the talal # of Dimes and Pennies.

How many of each does she have?

D → # Dimes

N → # Nickels

P → # Pennies

$D + N + P = 20$

$10D + 5N + 1P = 132$

$N = D + P$

$$\begin{cases} D + N + P = 20 \\ 10D + 5N + P = 132 \\ -D + N - P = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 10 & 5 & 1 & 132 \\ -1 & 1 & -1 & 0 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 10 & 5 & 1 & 132 \\ -1 & 1 & -1 & 0 \end{array}\right]$$

$(-10)R1 + R2 \to R2$

$R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & -5 & -9 & -68 \\ 0 & 2 & 0 & 20 \end{array}\right]$$

$R3 \div (2) \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & -5 & -9 & -68 \\ 0 & 1 & 0 & 10 \end{array}\right]$$

$R2 \leftrightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 1 & 0 & 10 \\ 0 & -5 & -9 & -68 \end{array}\right]$$

$(5)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & -9 & -18 \end{array}\right]$$

$R3 \div (-9) \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} \\ N=10 \\ P=2 \end{array}$$

$D + N + P = 20$

$D + 10 + 2 = 20$

$\boxed{D = 8}$

8 Dimes
10 Nickels, and
2 Pennies

Andre invested \$20,000 in 3 accounts for one year in Simple interest. He made \$620 in interest. Bank account @ 2%, Stocks @ 4%, Bonds @ 5%. He invested 5 times in the Bank account as Stocks.

$B \to$ Bank
$S \to$ Stocks
$D \to$ Bonds

$B + S + D = 20000$

$100\ \boxed{2\% B + 4\% S + 5\% D = 620}$

$B = 5S$

$$\begin{cases} B + S + D = 20000 \\ 2B + 4S + 5D = 62000 \\ B - 5S = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20000 \\ 2 & 4 & 5 & 62000 \\ 1 & -5 & 0 & 0 \end{array}\right]$$

$(-2)R1 + R2 \to R2$ $(-1)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20000 \\ 0 & 2 & 3 & 22000 \\ 0 & -6 & -1 & -20000 \end{array}\right]$$

$(3)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20000 \\ 0 & 2 & 3 & 22000 \\ 0 & 0 & 8 & 46000 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20000 \\ 0 & 2 & 3 & 22000 \\ 0 & 0 & 8 & 46000 \end{array}\right]$$

$R3 \div 8 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20000 \\ 0 & 2 & 3 & 22000 \\ 0 & 0 & 1 & 5750 \end{array}\right] \text{Bonds}$$

$B + S + D = 20000$

$B + 2375 + 5750 = 20000$

$B + 8125 = 20000$

$B = 11875$

$2S + 3D = 22000$

$2S + 3(5750) = 22000$

$2S = 4750$

$\boxed{S = 2375}$

Double check calculations

Bank → \$11875

Stocks → \$2375

Bonds → \$5750

SG 9

#10

P → Pizza

C → Ice Cream

S → Soda

Lisa $1P + 1C + 1S = 1030$

Mark $2P + 1C + 2S = 1910$

David $3P + 0C + 2S = 2420$

Your job is to solve for P, C, and S.

SG 8

#7

$4A + 6B + 4C = 30g$ Protein

$6A + 1B + 1C = 16g$ Fat

$3A + 1B + 12C = 24g$ Carb

Your job is to solve for A, B, and C.

Solve by elimination

$$\begin{cases} x^2 + \cancel{y^2} = 34 \\ x^2 - \cancel{y^2} = 16 \end{cases}$$

$2x^2 = 50$

$2x^2 = 50$

Divide by 2 $\Rightarrow x^2 = 25$

$x^2 + y^2 = 34$

$25 + y^2 = 34$

$y^2 = 34 - 25$

$y^2 = 9$

$\boxed{y = \pm 3}$

$\boxed{x = \pm 5}$

Final Ans

$\boxed{(5,3), (5,-3), (-5,3), (-5,-3)}$

$$\begin{cases} x^2 + y^2 = 25 \\ y = x + 1 \end{cases}$$

Solve by Subs. method

$y = x + 1$

$x = 3 \rightarrow y = 3 + 1 = 4$

$(3,4)$

$x = -4 \rightarrow y = -4 + 3 = -3$

$(-4,-3)$

$\{(3,4), (-4,-3)\}$

$x^2 + (x+1)^2 = 25$

$x^2 + (x+1)(x+1) \quad -25 = 0$

$x^2 + x^2 + x + x + 1 \quad -25 = 0$

$2x^2 + 2x - 24 = 0$

Divide by 2

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x + 4 = 0$
$x = -4$

$x - 3 = 0$
$x = 3$

Class QZ 11

Solve by matrix Method:

$$\begin{cases} x + 2y = 5 \\ 2x - y = 0 \end{cases}$$